8.3 Further Complex Numbers

Question Paper

Course	CIE A Level Maths
Section	8. Complex Numbers
Topic	8.3 Further Complex Numbers
Difficulty	Hard

Time allowed: 40

Score: /30

Percentage: /100

Question 1

Express the following complex numbers in exponential form:

(i) $3(2\cos 2 - 2\mathrm{i}\sin(-2))$

(ii) $-2 + 2\sqrt{3}\,\mathrm{i}$

[4 marks]

Question 2

$z_1 = 6\mathrm{e}^{4\mathrm{i}}$

$z_2 = 8\mathrm{e}^{-\mathrm{i}}$

(i) Work out $z_1 z_2$ and $\frac{z_1}{z_2}$, giving your answers in exponential form.

(ii) Express your answers to part (i) as complex numbers in modulus-argument form. In each case the modulus and argument should be given as exact values, with the argument θ being given in the interval $-\pi < \theta \leq \pi$.

[4 marks]

Question 3

Given the point z on an Argand diagram, where $z \neq 0$ is a complex number, describe the geometrical transformations that will map z to each of the following points:

(i) $-2z$

(ii) $|z|$

(iii) $\frac{z}{w}$ (where w is a non-zero complex number)

[6 marks]

Question 4a

Let $z = r(\cos\theta + \mathrm{i}\sin\theta)$ be a square root of the complex number $-5\sqrt{3} - 5\mathrm{i}$.

(a) Show that

$$r^2(\cos 2\theta + i\sin 2\theta) = 10\left(\cos\left(-\frac{5\pi}{6}\right) + \mathrm{i}\sin\left(-\frac{5\pi}{6}\right)\right)$$

[3 marks]

Question 4b

(b) Use the geometry of complex numbers to explain why

$$\cos\alpha + \mathrm{i}\sin\alpha = \cos(\alpha + 2\pi) + \mathrm{i}\sin(\alpha + 2\pi)$$

for any value of α, where α is a real number.

[2 marks]

Question 4c

(c) Use your answers to parts (a) and (b) to find the two square roots of the complex number $-5\sqrt{3} - 5\mathrm{i}$. Give your answers both in modulus-argument form and in the form $a + b\mathrm{i}$ where a and b are real numbers.

[4 marks]

Question 5

$$z = 3 + 3\sqrt{3}\,\mathrm{i}, \quad \mathrm{Re}(z^2 w) = 0, \quad |z^2 w| = 2|z|$$

Use geometrical reasoning to find the two possibilities for w, giving your answers in exponential form.

[4 marks]

Question 6

By considering the exponential and modulus-argument forms of a complex number, prove *Euler's identity*

$$\mathrm{e}^{\mathrm{i}\pi} + 1 = 0$$

[3 marks]